Stress analysis methods for underground pipe lines

Analysis of soil-pipe interaction involves investigation of soil forces, longitudinal/lateral pipe movement

Liang-Chaun Peng, Mechanical Engineer,
AAA Technology and Specialties Co., Inc., Houston

BECAUSE the major portion of a pipe line is normally buried, soil-pipe interaction analysis is the most important part of pipe line stress analysis. First, however, soil forces that are acting on the pipe must be investigated.

These forces differ somewhat from those encountered in foundation engineering problems. For instance, the often-referenced lateral pile loading data are hardly applicable to a pipe line problem since a pile is driven into the soil vertically rather than buried horizontally and lateral pile movement is much smaller than pipe line movement.

Fig. 5(a) shows a pipe line buried in a ditch. Because of the soil backfill and the pipe's own weight, the pipe receives a soil pressure acting at its surface as shown in Fig. 5(b). This pressure creates a bending stress on the pipe wall and at the same time produces a soil friction force against any axial pipe movement.

Except in highway or railroad crossings, the bending stress created by uneven soil pressure is negligible. If no casing is used at road crossings, bending stress due to soil pressure can be significant and should be evaluated using methods described by Spangler.[6] The code requires that this bending stress be combined with pressure hoop stress, and the combined stress should be limited to no more than the specified minimum yield strength (SMYS).

Axial friction force. Friction force is the first soil force that affects pipe movement. This section covers friction force that is created against the axial pipe movement.

Theoretically, friction force is equal to the product of the friction coefficient and the total normal force acting all around the pipe. Since actual distribution of normal force, Fig. 5(b), is hard to determine for the purpose of friction force calculation, a simplified model as shown in Fig. 5(c) can be used.

The normal force acting on the pipe surface can be divided into top force, W, and bottom force, $W + W_p$, where W_p is the weight of the pipe and its content. For a pipe buried in a ditch, the top soil force can be calculated by Marston's formula,[7] but in cases where the soil cover depth ranges from one to three times the pipe diameter, the force can be taken as the weight of the soil surcharge over the pipe. Hence, axial friction force can be written as:

$$f = \mu(W + W + W_p)/12$$

or

$$f = \mu(2\gamma DH + W_p)/12 \qquad (9)$$

in which, f = Axial friction force, lbs./in.
μ = Coefficient of friction between pipe and soil
γ = Density of backfill soil, lbs./ft.^{3}
D = Outside diameter of pipe, ft.
H = Depth of soil cover to top of pipe, ft.
W_p = Weight of pipe and content, lbs./ft.

The soil density and friction coefficient are obtained from soil tests performed along the pipe line route. In cases when test data are not available, the following

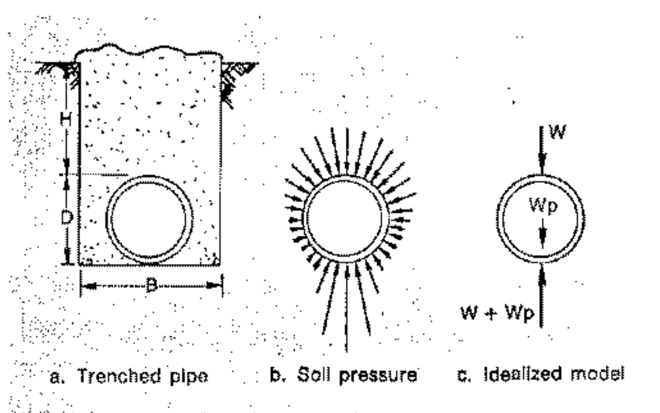

Fig. 5—Soil pressure distribution.

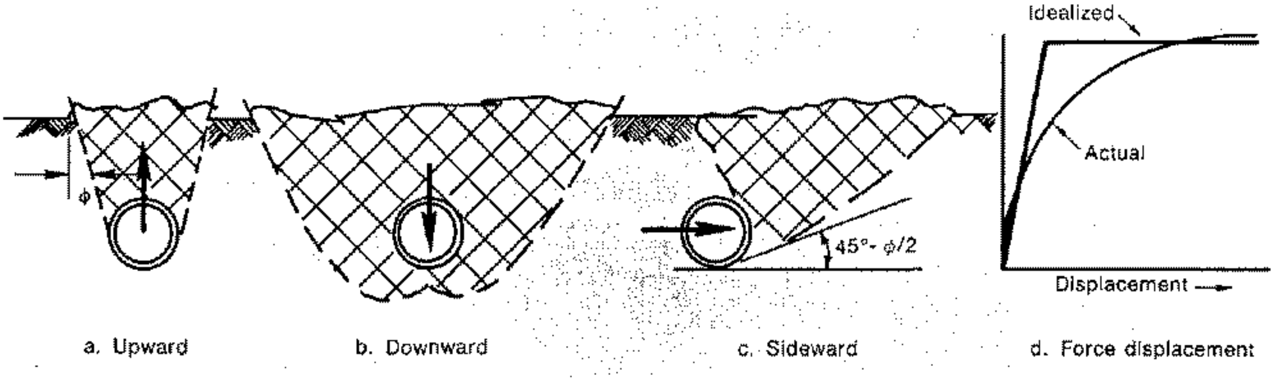

Fig. 6—Lateral soil forces.

friction coefficient can be used:[8]

Silt 0.3	Sand 0.4	Gravel 0.5.

The above coefficients are the lower bond values equivalent to the sliding friction. The static coefficient of friction can be as much as 70 percent higher.[9]

For pipe lines buried below the water table, buoyant force should be subtracted from soil and pipe weight before entering Equation 9 for calculation.

Lateral soil force. Fig. 6 shows three different lateral soil forces normally encountered in pipe line analysis. Each lateral force can be idealized, as in Fig. 6(d), into two stages: **Elastic stage,** where resistance force is proportional to pipe displacement, and **plastic stage,** where resistance remains constant regardless of displacement.

Though elastic constant can be evaluated directly by test or published methods,[10] they are generally very sensitive to the data gathered. An alternate method is to calculate from the more reliable ultimate resistance. Several authors have reported that displacement required to reach ultimate resistance is about 1.5 to 2 percent of the pipe bottom depth.[11]

From this important finding, elastic constant can be calculated from ultimate resistance by taking 1.5 percent of the total depth as yield displacement. Using 1.5 percent instead of 2 percent gives a more realistic secant modulus which will underestimate the modulus for initial displacement but somewhat overestimate the modulus at higher displacement. For a pipe line, underestimation of initial modulus is greatly compensated by the fact that less than perfect backfill compaction does provide initial softness.

When a pipe moves horizontally as in Fig. 6(c), it creates a passive soil pressure at the front surface, and at the same time receives an active soil force at the back. Because of the arch action, a void will be created behind the pipe as soon as it moves a small distance and the active soil force can therefore be disregarded.[10] The only lateral force is the passive soil force which can be written as:

$$U = \frac{1}{2}\gamma(H+D)^2 \tan^2\left(45 + \frac{\phi}{2}\right) \qquad (10)$$

Where U is the ultimate soil resistance, lbs./ft., and the other symbols are as previously defined. Strictly speaking, Equation 10 is valid only when the soil cover, H, is less than the pipe diameter, D. It will overestimate the resistance for deeper soil covers. However, for a three-diameter deep cover with dense granular soil, the overestimation is only about 10 percent.[12] This is within the variation of other parameters involved in soil mechanics.

Taking 1.5 percent of the total depth as the yield displacement, the elastic constant can be written as:

$$K = \frac{U}{0.015\,(H+D)\times 144}$$

$$= 0.2315\,\gamma\,(H+D)\,\tan^2\left(45 + \frac{\phi}{2}\right) \qquad (11)$$

Where the elastic constant, K, in pounds per inch of pipe per inch displacement, is the product of modulus of passive resistance and pipe diameter. It should be noted that instead of determining soil modulus ϵ, the constant ϵD is determined. This is similar to the ϵR constant used in the famous Iowa flexible pipe deflection formula.[6]

Longitudinal pipe movement. The flexibility problem originates from the expansion of the pipe. Therefore, the first step of flexibility analysis is to determine longitudinal movement.

Fig. 7 shows a pipe line leaving a pump station. Point A is a scraper launching barrel and BCD represents a very long line. When the line is heated up, the end of pipe B will start to move. The movement produces friction force, f, while at the same time an end resistance, Q,

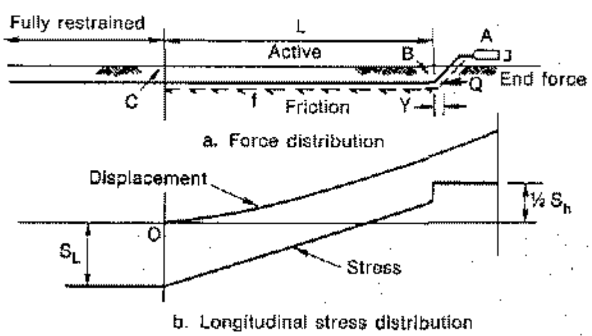

Fig. 7—Longitudinal movement.

develops because of soil passive force and pipe stiffness. The moving portion of the pipe will extend gradually downstream to a point C where the movement stops.

As the moving portion extends, friction force also increases, and when the moving boundary reaches point C, friction force plus end force developed is enough to suppress the expansion completely. Point C is sometimes called virtual anchor point and the moving length, L, the active length.

Because of the unequal amount of friction force received, longitudinal stress along the active length varies from point to point. Distribution of longitudinal stress is shown in Fig. 7(b). At the scraper barrel end, the stress is tensile and equal to the pressure stress. The tensile stress is reduced gradually due to end force and friction force, then eventually becomes compressive if the line is hot enough. Finally, at point C, the compressive stress reaches maximum and stays the same for the entire fully restrained portion.

The active length of the line can be determined by equating friction force plus end force with the required anchor force obtained from Equation 6, that is:

$$fL + Q = F$$

or

$$L = \frac{F - Q}{f} \qquad (12)$$

where L = Active length, in.
F = Anchor force or expansion force, lbs.
Q = End resistance force, lbs.
f = Soil friction force, lbs./in.

After the active length is determined, the end movement, y, can be calculated by multiplying the average expansion rate with the length. The expansion rate at C is zero, and the rate at end B is equivalent to the pull of the potential expansion force (or anchor force) minus end force, hence:

$$y = \frac{1}{2}\left[\ 0.0 + \frac{1}{AE}(F - Q)\right]L$$

substituting Equation 12 we have:

$$y = \frac{1}{2AEf}(F - Q)^2 \qquad (13)$$

where y is the end deflection in inches.

The end deflection is proportional to the square of the net expansion force. The underground piping is therefore nonlinear and cannot be solved by direct linear simulation.

Lateral pipe movement. The lateral pipe movement is caused by longitudinal movement of a pipe connected in the perpendicular direction.

Fig. 8(a) shows a long main line pipe making a 90-degree turn to enter a pump station. Expansion of the long pipe AB has caused the station pipe BC to move in the lateral direction. The lateral movement at corner B

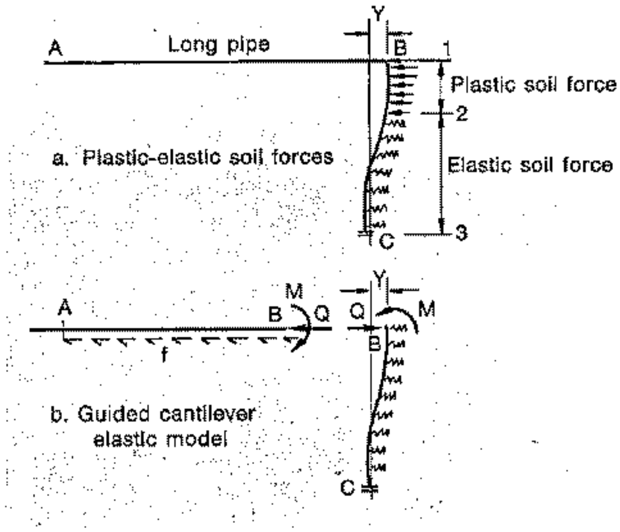

Fig. 8—Lateral movement.

is y inch and decreases gradually toward point C where displacement is virtually zero.

Because of the large movement, the soil in region 1-2 is in plastic stage offering constant passive force. The soil in region 2-3 is still in the static range that offers a resisting force proportional to local displacement. The extent of region 1-2 depends on the magnitude of end movement, and is nonexistent for some low temperature lines.

The analysis involving elastic-plastic soil force generally requires step-wise linear computer simulations. The piping, however, can be conservatively treated as a guided cantilever elastic system which can be easily analyzed.

As shown in Fig. 8(b), the long pipe AB is considered to be guided, allowing no rotation at the corner B. The soil force is considered to be perfect elastic, offering resistance proportional to the local displacement. This approach tends to underestimate the deflection because of the large soil force and stiff boundary assumed.

On the contrary, the method will tend to overestimate the moment because of the stiff nonrotational corner B assumed. Since stress is determined by the moment, it is apparent that the aproach is conservative.

To start the analysis, the system is cut into two free bodies as shown in the figure. The long pipe AB is exactly the same as shown in Fig. 7(a) except the end moment, M. Since the end moment does not affect the longitudinal movement, we still can say:

$$y = \frac{1}{2AEf}(F - Q)^2$$

Here we have one equation but two unknowns, y and Q. Therefore, another equation is needed from leg BC before the problem can be solved.

The situation in leg BC is a beam on elastic foundation problem. The case is not quite the same as an ordinary pile problem where elastic modulus changes with depth and the end, in most cases, is free to rotate. The pile formula, especially those that do not produce any end moment, cannot be used.

Leg BC actually represents one-half of an infinite beam on elastic foundation that is loaded with a concentrated force. From literature citation 14, we can write:

$$y = \frac{Q\beta}{K} \qquad (14)$$

$$M = \frac{Q}{2\beta} \qquad (15)$$

in which, y = End displacement, in.
Q = End force, lbs.
K = Soil elastic constant, lbs. in.^{2}
E = Modulus of elasticity of pipe, psi
I = Moment of inertia of pipe, in.^{4}
M = End bending moment, in.-lbs.

$$\beta = \sqrt[4]{\frac{K}{4EI}}$$

Substituting Equation 13 in Equation 14 and rearranging the form, we have:

$$Q = C - \sqrt{C^2 - F^2} \qquad (16$$

where

$$C = F + \frac{\beta AEf}{K}$$

After the end force is determined, the end displacement and moment can be calculated from Equations 14 and 15, respectively.

Sample calculations. Assume the same 20-inch diameter pipe, described in Part 1, is buried with 4 feet of soil cover and the soil is silty sand with a density of 125 lbs./ft.^{3} and an internal friction angle of 30 degrees. The displacement and stress of the pipe shown in Fig. 7 and Fig. 8 can then be calculated as follows:

1. Soil friction force. As discussed earlier, a sliding friction factor of 0.4 can be used for the silty sand against the pipe. Assuming the specific weight of the crude is 0.85, the friction force from Equation 9 is:

$$f = 0.4 \times (2 \times 125 \times \frac{20}{12} \times 4 + 185.7)/12 = 61.74 \text{ lbs./in.}$$

2. Soil end force Q **acting on the vertical entry leg of** Fig. 7 can be calculated by adding side shears to Equation 10.[15] That is,

$$Q = \frac{\gamma}{2}(H+D)^2 \tan^2\left(45 + \frac{\phi}{2}\right)D + \frac{H+D)^3 \gamma K_o \tan\phi}{3\tan(45+\phi/2)}$$

$$= \frac{125}{2}\left(4 + \frac{20}{12}\right)^2 \tan^2(60) \times \frac{20}{12} + \frac{\left(4+\frac{20}{12}\right)^3 \times 125 \times 0.5\tan(30)}{3\tan(60)}$$

$$= 11{,}296 \text{ lbs.}$$

in which $K_o = 0.5$ is the coefficient of later soil pressure.

3. Active length is calculated by Equation 12 as:

$$L = \frac{F-Q}{f} = \frac{706280 - 11296}{61.74} = 11{,}256 \text{ in.} = 938 \text{ ft.}$$

Expansion force $F = 706280$ is calculated in Part 1.

4. Longitudinal movement at the scraper barrel of Fig. 7 can be found from Equation 13:

$$y = \frac{1}{2 \times 23.1 \times 27.9 \times 10^6 \times 61.74} (706280 - 11296)^2 = 6.07 \text{ in.}$$

which is only slightly smaller than the free end ($Q = 0$) expansion of 6.27 inches. In reality the movement will be consideraby smaller due to the lateral soil force acting on the station pipe as will be shown in the following. The slacks in the main line will also absorb part of the movement.

5. Lateral soil force and elastic constant are calculated by Equations 10 and 11, respectively.

$$U = \frac{1}{2} \times 125\left(4 + \frac{20}{12}\right)^2 \tan^2(45+15) = 6020 \text{ lbs./ft.}$$

$$K = 0.2315 \times 125\left(4 + \frac{20}{12}\right)\tan^2(45+15)$$

$$= 491.4 \text{ lbs./in.}^2$$

6. Then for the Fig. 8 pipe we have:

$$\beta = \sqrt[4]{\frac{K}{4EI}} = \sqrt[4]{\frac{491.4}{4 \times 27.9 \times 10^6 \times 1110}}$$

$$= 0.0079 \text{ in.}^{-1}$$

$$C = F + \frac{\beta AEF}{K} = 706280 + \frac{0.0079 \times 23.1 \times 27.9 \times 10^6 \times 61.74}{491.4}$$

$$= 1346499 \text{ lbs.}$$

End force $Q = C - \sqrt{C^2 - F^2} = 1346499 - \sqrt{1346499^2 - 706280^2} = 200101$ lbs.

End displacement, $y = \frac{Q\beta}{K} = \frac{200101 \times 0.0079}{491.4} = 3.22$ in.

This displacement greatly exceeds the yield displacement of $0.015\ (H+D) = 0.085$ ft. = 1.02 in., therefore the calculation is only a rough estimation. A more accurate analysis will require the consideration of the plastic soil force. The end moment is calcuated by Equation 15:

$$M = \frac{Q}{2\beta} = \frac{200101}{2 \times 0.0079} = 12664620 \text{ in.-lbs.}$$

The bending stress without considering stress intensification is:

$$S = \frac{M}{Z} = \frac{12664620}{111} = 114{,}095 \text{ psi}$$

which is about three times the allowable of 37,440 psi.

Although a step-wise computer simulation might reduce the stress somewhat, some remedies are still required.

Part 2 conclusion. From the above discussion, it can be concluded that:

- For a buried pipe line, the pipe will expand toward the end or a bend, but the central portion of the line will be fully restrained by the soil friction force. Total movement at the free end is inversely proportional to soil friction force but is directly proportional to the square of the temperature difference between operating and installation conditions.

- Because of the lateral soil force, movement at a bend is about one-half of movement at the free end.

- For 20-inch standard pipe at 130° F temperature difference, stress developed at the bend connecting to a long run is about three times the allowable stress. Therefore proper care should be taken to reduce the stress. The most often used methods are:

  (a) Install an anchor at about 20-diameter length away from the bend to reduce the movement.

  (b) Install soft material behind the pipe of the lateral leg.

  (c) Locally use thicker wall pipe near the bend area.

  (d) Adopt special backfilling procedure.

- A buried pipe bend without any particular attention will take only about 60° F temperature raise if it is connected to a long run of pipe.

The techniques developed in this article are generally sufficient to handle the routine analyses. However, there are configurations that would require computerized step-wise linear simulation.

In summary, the most difficult part of the soil pipe interaction analysis is to determine the soil characteristics. Once the soil data are available, the analysis can be performed routinely. Unfortunately, we still lack reliable soil correlation formulas that can be used confidently by the analysts. Although the two soil formulas presented in this article are simplified, they do give reasonable numbers. The most important thing they provide is clear physical pictures of the moving processes. By relating one's thinking to a physical picture, it is less likely that an extreme value will be used.

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Fig 8 Example can be simulated and analyzed easily using SIMFLEX program.

Download a Demo Version and try a 6" Sh-40 pipe as follows.

```
UNDERGROUND PIPELINE
OPTION, CODE=4, PELONG, TW, FRICT
SPIPE1, D=-6, THK=S40, TEMP=150
        P=500, CSG=0.8, MATL=CS
5, LONGPIPE(250), PIPE1.
10, X=60, RSEC=10,
    STY, NLZ (1200, 0.8), BR
15, Z=80, RSEC=12
    NLY ( , , 0.4), NLX (1400, 0.8)
20, Z=10, ANCH
END
```

TRY ABOVE DATA TO GET A FEELING.